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QE Approach to Common Lyapunov Function Problem

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Abstract

This paper introduces an application of the Quantifier Elimination (QE) method to common Lyapunov function problem. The common Lyapunov function problem is a problem that concerns with the existence of a common Lyapunov function for a set of linear systems. This problem is basic to the system stability problem. In this paper, we present that by applying QE method, a symbolic necessary and sufficient existence condition of a common quadratic Lyapunov function for a pair of second-order systems can be obtained. For a set of second-order systems, the existence condition also can be identified by QE method. For the case of common infinity-norm Lyapunov functions, sufficient conditions can be obtained by applying QE method to sufficient existence conditions. Several numerical examples are given to illustrate the method.

1 Introduction

The common Lyapunov function problem is a problem that investigates the existence of a common Lyapunov function for a set of linear time-invariant systems. The problem frequently arises in stability analysis and control design of various types of control systems such as uncertain systems [2], fuzzy systems [15], switched systems [7] etc. Presently, there are two main types of common Lyapunov functions: common quadratic Lyapunov function (CQLF) and common infinity-norm Lyapunov function (CILF). The CQLF and CILF problems are studied in [8, 9, 11, 13, 14] and [8, 9, 10, 12] respectively. One important issue of the common Lyapunov function problem is its existence conditions. Currently, with

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the help of LMI packages, given a set of stable constant linear systems, we can identify whether the systems share a CQLF and construct such CQLF. With CILF, [12] provides an algorithm to construct a CILF. However, since the algorithm is based on a sufficient existence condition of a infinity-norm Lyapunov function, we cannot always check and construct a CILF. We are concerned with the symbolic existence condition problem, i.e., the problem of finding symbolic existence conditions on system matrices such that these systems share a common Lyapunov function. The meaning of this problem is that the symbolic existence conditions give us stability region of parameters for control systems. In the research of this problem, to the best of the authors' knowledge, so far, the methods to find CQLF symbolic existence conditions for a pair of second order systems is shown only in [11]. However, the method in [11] requires so intensive computation that only trivial problems of a single system parameter can be solved. A CQLF existence condition for a set of second order systems is given in [13], but the condition also suffers computation complexity and generally cannot solve the symbolic existence condition problem. For CILF, some methods available at present are shown only in [10] to identify symbolic sufficient existence conditions.

QE method is a mathematical method used to solve the multivariate polynomial inequalities (MPIs) with quantifiers and quantified variables. In QE method, by eliminating the quantifiers such as "for all ($\forall$)" and "there exists ($\exists$)", one can receive symbolic results in the form of quantifier-free equalities and/or inequalities or logical output such as "true" or "false". QE method has various applications in different scientific and engineering fields. In the control theory and control engineering, QE method is successfully applied to solving problems of robust control, stability analysis etc. [1, 4]. The CQLF and CILF problems can be expressed in the forms of MPIs with quantifiers and quantified variables. Therefore the application of QE method to the common Lyapunov function problem is a feasible direction, which has not been studied so far. In this paper, the application of QE method to the CQLF and CILF problems is introduced. The merit of QE method here is that for CQLF, comparing to the methods in [11, 13], it can solve the problem for a pair of second order systems with two system parameters and the problem for a set of second order systems. Moreover, for CILF, QE method can give larger existence regions than the methods presented in [10]. Several examples are given to illustrate QE method. The content of this paper is organized as follows: Section 2 presents basic definitions in CQLF and CILF problems. Section 3 presents the application of QE method to the CQLF and CILF problems and gives some numerical examples. Conclusion is given in Section 4.

2 Common Lyapunov Function Problem

First, we presents basic definitions of CQLF [8, 9] and CILF [5, 8]. We consider a set of continuous-time linear time-invariant systems described by the equations

$$\dot{\mathbf{x}} = \mathbf{A}_{ci}\mathbf{x}, \mathbf{x} \in \mathcal{R}^n, \mathbf{A}_{ci} \in \mathcal{R}^{n\times n}, i = 1, ..., q, \tag{1}$$

and a set of discrete-time linear time-invariant systems described by the equations

$$\mathbf{x}(t+1) = \mathbf{A}_{di}\mathbf{x}(t), t = 0, 1, ...; \mathbf{x} \in \mathcal{R}^n, \mathbf{A}_{di} \in \mathcal{R}^{n\times n}, i = 1, ..., q. \tag{2}$$

Definition 1. The set of systems (1) is said to have a CQLF if there exists a symmetrical positive definite matrix $\mathbf{P} = \mathbf{P}^T > 0$ such that the following Lyapunov inequalities

$$\mathbf{P}\mathbf{A}_{ci} + \mathbf{A}_{ci}^T\mathbf{P} < 0, \forall i = 1, ..., q \tag{3}$$

are satisfied and the CQLF is $\mathbf{V}(\mathbf{x}) = \mathbf{x}^T\mathbf{P}\mathbf{x}$.

Definition 2. The set of systems (2) is said to have a CQLF if there exists a symmetrical positive definite matrix $\mathbf{P} = \mathbf{P}^T > 0$ such that the following Lyapunov inequalities

$$\mathbf{A}_{di}^T\mathbf{P}\mathbf{A}_{di} - \mathbf{P} < 0, \forall i = 1, ..., q \tag{4}$$

are satisfied and the CQLF is $\mathbf{V}(\mathbf{x}) = \mathbf{x}^T\mathbf{P}\mathbf{x}$.

Definition 3. The function of the vector norm form $\mathbf{V}(\mathbf{x}) = ||\mathbf{W}\mathbf{x}||_\infty, \mathbf{W} \in \mathcal{R}^{m\times n}$, $m \geq n$, $\text{rank}(\mathbf{W}) = n$ is said to be a CILF for the set of systems (1) if there exist matrices $\mathbf{Q}_i \in \mathbf{R}^{m\times m}, i = 1, ..., q$ such that we have the matrix relations

$$\mathbf{W}\mathbf{A}_{ci} = \mathbf{Q}_i\mathbf{W} \tag{5}$$

$$\mu_\infty(\mathbf{Q}_i) < 0, i = 1, ..., q. \tag{6}$$

Here $\mu_\infty(\mathbf{Q})$ is the infinity-norm matrix measure of the matrix $\mathbf{Q} = \{q_{ij}\} \in \mathcal{R}^{m\times m}$, defined by

$$\mu_\infty(\mathbf{Q}) = \max_{1\leq i\leq m}\{q_{ii} + \sum_{j=1, j\neq i}^{m} |q_{ij}|\}. \tag{7}$$

Definition 4. The function of the vector norm form $\mathbf{V}(\mathbf{x}) = ||\mathbf{W}\mathbf{x}||_\infty, \mathbf{W} \in \mathcal{R}^{m\times n}$, $m \geq n$, $\text{rank}(\mathbf{W}) = n$ is said to be a CILF for the set of systems (2) if there exist matrices $\mathbf{Q}_i \in \mathbf{R}^{m\times m}, i = 1, ..., q$ such that we have the matrix relations

$$\mathbf{W}\mathbf{A}_{di} = \mathbf{Q}_i\mathbf{W} \tag{8}$$

$$||\mathbf{Q}_i||_\infty < 1, i = 1, ..., q. \tag{9}$$

Here $||\mathbf{Q}||_\infty$ is the infinity-norm of the matrix $\mathbf{Q} = \{q_{ij}\} \in \mathcal{R}^{m\times m}$, obtained through

$$||\mathbf{Q}||_\infty = \max_i\{\sum_{j=1}^{m} |q_{ij}|\}. \tag{10}$$

Concerning the CQLF problem for the systems (1) and (2), we have a known result stated as follows [6]:

Theorem 1: Suppose the systems (1) and (2) have the following matrix transformation relations

$$\mathbf{A}_{di} = (\mathbf{I}_n - \mathbf{A}_{ci})^{-1}(\mathbf{I}_n + \mathbf{A}_{ci}) \tag{11}$$

or equivalently

$$\mathbf{A}_{ci} = (\mathbf{A}_{di} + \mathbf{I}_n)(\mathbf{A}_{di} - \mathbf{I}_n)^{-1}. \tag{12}$$

If the systems (1) have a CQLF then the systems (2) have a CQLF and vice-verse, and in this case the systems (1) and (2) have the same CQLF.

In the CQLF and CILF symbolic existence condition problems, the unquantified variables are the entries of system matrices $\mathbf{A}_{ci}$ and $\mathbf{A}_{di}$ and the quantified variables are the entries of component matrices $\mathbf{P}$ and $\mathbf{W}$ of CQLF and CILF. In case of no unquantified variable, i.e., given a set of constant linear systems, QE method also can be applied to check the existence of a CQLF or CILF. In the next section, QEPCAD software [3] is used to solve the CQLF and CILF problems.

3 Solution by QE Method to Common Lyapunov Function Problem

3.1 CQLF Problem

3.1.1 Pair of Second-Order Systems

We consider two Hurwitz stable second-order continuous-time linear time-invariant systems described by the following equations

$$\dot{\mathbf{x}} = \mathbf{A}_{ci}\mathbf{x}, \mathbf{x} \in \mathcal{R}^2, \mathbf{A}_{ci} \in \mathcal{R}^{2\times 2}, i = 1, 2. \tag{13}$$

QE method can be applied to Lyapunov matrix inequalities (3) to find the condition on system matrices $\mathbf{A}_{c1}, \mathbf{A}_{c2}$ with two quantified variables on matrix $\mathbf{P}$. However, to reduce intensive computation, we use a known simpler result from [14] containing only a single quantified variable stated as follows:

Theorem 2. A necessary and sufficient condition for the two second-order systems (13) to have a CQLF is

$$\mathrm{Re}(\lambda(\mathrm{co}(\mathbf{A}_{c1}, \mathbf{A}_{c2}))) < 0 \tag{14}$$

$$\mathrm{Re}(\lambda(\mathrm{co}(\mathbf{A}_{c1}, \mathbf{A}_{c2}^{-1}))) < 0. \tag{15}$$

Here co(.) denotes convex hull (polytope) of matrices: $\mathrm{co}(\mathbf{X}, \mathbf{Y}) = \{\alpha\mathbf{X} + (1-\alpha)\mathbf{Y} : \alpha \in [0,1]\}$, $\lambda(\mathbf{X})$ denotes the eigenvalues of matrix $\mathbf{X}$ and Re(.) denotes the real part of a complex number.

Given a pair of second-order systems (13), QE method to identify the existence condition of a CQLF is expressed in the following order:

Step 1. Calculate matrices $\mathrm{co}(\mathbf{A}_{c1}, \mathbf{A}_{c2})$ and $\mathrm{co}(\mathbf{A}_{c1}, \mathbf{A}_{c2}^{-1})$.

Step 2. Find the characteristic equations to calculate the eigenvalues of matrices $\mathrm{co}(\mathbf{A}_{c1}, \mathbf{A}_{c2})$ and $\mathrm{co}(\mathbf{A}_{c1}, \mathbf{A}_{c2}^{-1})$.

Step 3. Express the condition $\mathrm{Re}(\lambda(\mathrm{co}(\mathbf{A}_{c1}, \mathbf{A}_{c2}))) < 0$ and $\mathrm{Re}(\lambda(\mathrm{co}(\mathbf{A}_{c1}, \mathbf{A}_{c2}^{-1}))) < 0$ $\forall\alpha \in [0,1]$. This can be done by applying the Liènard Chipart criterion to the polynomials obtained in **Step 2**.

Step 4. Apply QE software to identify the range of entries in the matrices $\mathbf{A}_{c1}, \mathbf{A}_{c2}$.

We give an example as follows:

Example 1: Let $\mathbf{A}_{c1}, \mathbf{A}_{c2}$ be given by

$$\mathbf{A}_{c1} = \begin{bmatrix} x & 0 \\ y & -1 \end{bmatrix}, \mathbf{A}_{c2} = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}, \tag{16}$$

where $x, y \in \mathcal{R}$ are system parameters. We wish to identify x, y so that the systems (16) have a CQLF.

The method in [11] is unable to deal with the example with two parameters due to complicated calculation. But by applying QE method with the above steps, the problem can be solved. After calculating $\mathrm{co}(\mathbf{A}_{c1}, \mathbf{A}_{c2})$ and $\mathrm{co}(\mathbf{A}_{c1}, \mathbf{A}_{c2}^{-1})$, the CQLF existence condition can be expressed in the following quantified polynomial inequalities satisfying $\forall\alpha \in [0,1]$:

$$\begin{cases} 2 - \alpha - x\alpha > 0 \\ 1 - 2\alpha - 2x\alpha - y\alpha + \alpha^2 + x\alpha^2 + y\alpha^2 > 0 \\ 1 + y\alpha - \alpha^2 - x\alpha^2 - y\alpha^2 > 0. \end{cases} \tag{17}$$

Applying the QEPCAD software to (17), the output turns out to be

$$\begin{cases} x < 0 \\ -2 - 2\sqrt{-x} < y < 2\sqrt{-x} - 2x. \end{cases} \tag{18}$$

The CQLF existence conditions (18) are the shaded region on $x - y$ coordinates in Fig. 1.

3.1.2 Set of Second-Order Systems

We now consider the following set of stable second-order continuous-time linear time-invariant systems

$$\dot{\mathbf{x}} = \mathbf{A}_{ci}\mathbf{x},\ \mathbf{x} \in \mathcal{R}^2,\ \mathbf{A}_{ci} \in \mathcal{R}^{2\times 2},\ i = 1, ..., q. \tag{19}$$

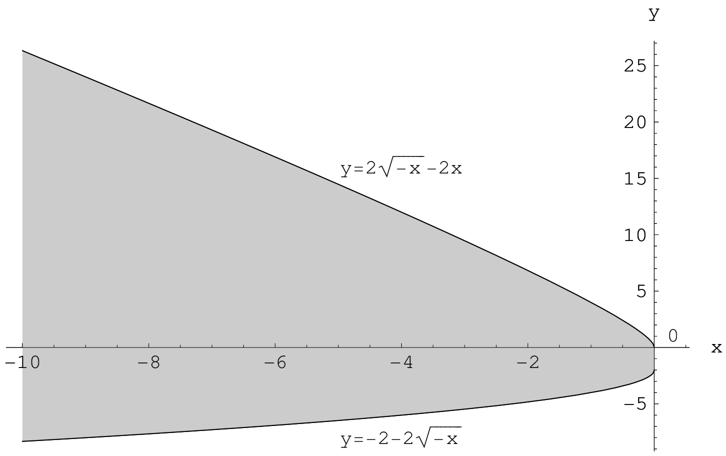

Figure 1: CQLF Existence Region

We can apply QE method directly to the Lyapunov matrix inequalities (3) to find symbolic existence conditions. However, to reduce the intensive computation, we use the following theorem:

Theorem 3: A necessary and sufficient condition for the set of systems (19) to share a CQLF is that there exists a matrix

$$\mathbf{P} = \begin{bmatrix} 1 & p \\ p & p_{22} \end{bmatrix} \tag{20}$$

such that

$$\det(\mathbf{P}\mathbf{A}_{ci} + \mathbf{A}_{ci}^T\mathbf{P}) > 0, \forall i = 1, ..., q \tag{21}$$

are satisfied.

The proof of this theorem is shown in [9]. Theorem 3 converts the problem from Lyapunov matrix inequalities (3) to algebraic inequalities (21), thus reduces the required computation. Given a set of stable second-order systems (19), QE method can be expressed in the following steps:

Step 1: Find the stability condition of system matrices $\mathbf{A}_{ci}, i = 1, ..., q$.

Step 2: Calculate matrices $\mathbf{P}\mathbf{A}_{ci} + \mathbf{A}_{ci}^T\mathbf{P}$ for $i = 1, ..., q$ with $\mathbf{P}$ in (20).

Step 3: Express inequalities

$$\det(\mathbf{P}\mathbf{A}_{ci} + \mathbf{A}_{ci}^T\mathbf{P}) > 0, i = 1, ..., q. \tag{22}$$

Step 4: Apply QE software to the inequalities (22) with conditions $\exists p, p_{22}$ to identify the condition on entries of system matrices $\mathbf{A}_{ci}$. Combine with the stability conditions in **Step 1**, we obtain the CQLF existence condition.

We take the following example:

Example 2: Consider a set of three second-order systems with system matrices

$$\mathbf{A}_{c1} = \begin{bmatrix} 0 & 5 \\ -30 & x \end{bmatrix}, \mathbf{A}_{c2} = \begin{bmatrix} 0 & 5 \\ -26 & -1 \end{bmatrix}, \mathbf{A}_{c3} = \begin{bmatrix} -6 & 27 \\ -150 & -1 \end{bmatrix}, \tag{23}$$

where $x \in \mathcal{R}$ is a system parameter. We now identify x such that the set of systems (23) share a CQLF.

With the example, while the method in [13] is unable to solve due to computational complexity, QE method is applicable. The stability condition is $x < 0$. We calculate $\mathbf{PA}_{ci} + \mathbf{A}_{ci}^T\mathbf{P}$ with $i = 1, 2, 3$ and express the CQLF existence condition in the quantified inequalities as follows: $\exists p, p_{22}$ such that

$$\begin{cases} -25 - 600p^2 + 300p_{22} - 900p_{22}^2 - 10px - 60pp_{22}x - p^2x^2 > 0 \\ -25 + 10p - 521p^2 + 260p_{22} + 52pp_{22} - 676p_{22}^2 > 0 \\ -729 - 270p - 16249p^2 + 8124p_{22} - 1500pp_{22} - 22500p_{22}^2 > 0. \end{cases} \tag{24}$$

Applying QEPCAD to (24), we get the result

$$5x + 7 \leq 0 \text{ or } 2601x^4 + 626760x^3 - 385050x^2 - 441000x + 1500625 < 0 \tag{25}$$

or equivalently $x < -1.31225$ (approximated value).

In the general case of a set of second order systems, the number of input formulas may exceed the ability of QEPCAD. In this case, the problem can be simplified by the help of the following theorem from [13]:

Theorem 4. A necessary and sufficient condition for the set of systems (19) to have a CQLF is that a CQLF exists for every 3-tuple of systems $(\mathbf{A}_{ci}, \mathbf{A}_{cj}, \mathbf{A}_{ck}), i \neq j \neq k$ for all $i, j, k \in \{1, ..., q\}$.

Theorem 4 shows that for the problem of a set of second-order systems, firstly we find symbolic existence conditions for every three systems by QE method as shown above, then combine these symbolic conditions to obtain the existence conditions for the systems (19) to share a CQLF. For the case of a set of second order discrete-time systems, by using matrix transformation in Theorem 1, the problem can be transformed to the case of a set of second-order continuous-time systems.

3.2 CILF Problem

Definitions 3 and 4 give us a necessary and sufficient condition of the existence of a CILF but the size m of the matrix $\mathbf{W}$ is not specified. We therefore restrict ourselves to the case when $\mathbf{W}$ is a square matrix, that is $m = n$, and apply QE method. However the problem is still not easy even for a simple case of a pair of second-order systems with two parameters in system matrices and one quantified variable in the matrix $\mathbf{W}$ ($\mathbf{W} \in \mathcal{R}^{2\times 2}$

contains three constants and one quantified variable). In order for QE method to be applicable, for the case of continuous-times systems, we use the following theorem from [10]:

Theorem 5: Given a nonsingular matrix $\mathbf{U} \in R^{n\times n}$, a necessary and sufficient condition for the existence of $\mathbf{D} = \text{diag}\{d_1, ..., d_n\}$ $(d_i > 0, i = 1, ..., n)$ such that $V(\mathbf{x}) = ||\mathbf{D^{-1}Ux}||_\infty$ to be a CILF for the set of systems (1) is that the simultaneous inequalities $\mathbf{B}_{ci}\mathbf{d} > 0$ $(i = 1, ..., q; \mathbf{d} = \{d_1, ..., d_n\}^T \in \mathcal{R}^n)$ have a common solution $\mathbf{d} > 0$, where $\mathbf{B}_{ci} = \{b^i_{cjk}\}$ is defined by

$$\begin{cases} b^i_{cjj} = -\tilde{a}^i_{cjj} \\ b^i_{cjk} = -|\tilde{a}^i_{cjk}|, (j \neq k), \end{cases} \tag{26}$$

and $\tilde{\mathbf{A}}_{ci} = \{\tilde{a}^i_{cjk}\} = \mathbf{U}^{-1}\mathbf{A}_{ci}\mathbf{U}$. Here for a vector $\mathbf{x} = \{x_1, ..., x_n\}^T \in \mathcal{R}^n$, $\mathbf{x} > 0$ denotes $x_i > 0, i = 1, ..., n$.

In fact, Theorem 5 is a sufficient existence condition of a CILF for the class of component matrices $\mathbf{W} = \mathbf{D}^{-1}\mathbf{U} \in R^{n\times n}$, where $\mathbf{D} = \text{diag}\{d_1, ..., d_n\}$ $(d_i > 0, i = 1, ..., n)$ and $\mathbf{U} \in R^{n\times n}$. Based on Theorem 5, QE method to identify the symbolic existence condition of a CILF can be expressed in the following order:

Step 1: For a given constant matrix $\mathbf{U}$, compute matrices $\tilde{\mathbf{A}}_{ci}$ for $i = 1, ..., q$.

Step 2: Compute matrices $\mathbf{B}_{ci}$ for $i = 1, ..., q$.

Step 3: Express the simultaneous inequalities $\mathbf{B}_{ci}\mathbf{d} > 0$.

Step 4: Apply QE software to the above simultaneous inequalities with conditions $\exists d_i > 0$ $(i = 1, ..., n)$ to find the existence condition with respect to entries in the matrices $\mathbf{A}_{ci}$.

Example 3: Let us take a numerical example of a pair of second-order systems with system matrices $\mathbf{A}_{c1}$ and $\mathbf{A}_{c2}$ being

$$\mathbf{A}_{c1} = \begin{bmatrix} x & 0 \\ y & -1 \end{bmatrix}, \mathbf{A}_{c2} = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}, \tag{27}$$

where $x, y \in \mathcal{R}$ are system parameters. We wish to identify the existence condition of a CILF $\mathbf{V}(\mathbf{x}) = ||\mathbf{D^{-1}Ux}||_\infty$ for the systems (27).

We select matrix

$$\mathbf{U} = \begin{bmatrix} 0 & -1 \\ -1 & -1 \end{bmatrix}, \tag{28}$$

and calculate $\tilde{\mathbf{A}}_{ci} = \mathbf{U}^{-1}\mathbf{A}_{ci}\mathbf{U}$ and $\mathbf{B}_{ci}$ for $i = 1, 2$. The quantified inequalities are: $\exists d_1, d_2$ such that

$$\begin{cases} d_1 > 0 \\ d_2 > 0 \\ d_1 - d_2 > 0 \\ d_1 + d_1 y - d_2|y| > 0 \\ -d_2 x - d_2 y - d_1|x + y + 1| > 0. \end{cases} \tag{29}$$

Applying QEPCAD software to (29) with condition $\exists\, d_1, d_2$ and simplifying the QEPCAD-generated result, the condition is

$$\begin{cases} x < 0 \\ (-1 - x - \sqrt{1+x^2})/2 < y < (-1-2x)/2. \end{cases} \tag{30}$$

The CILF sufficient existence condition (30) are the shaded region on $x - y$ coordinates shown in Fig. 2.

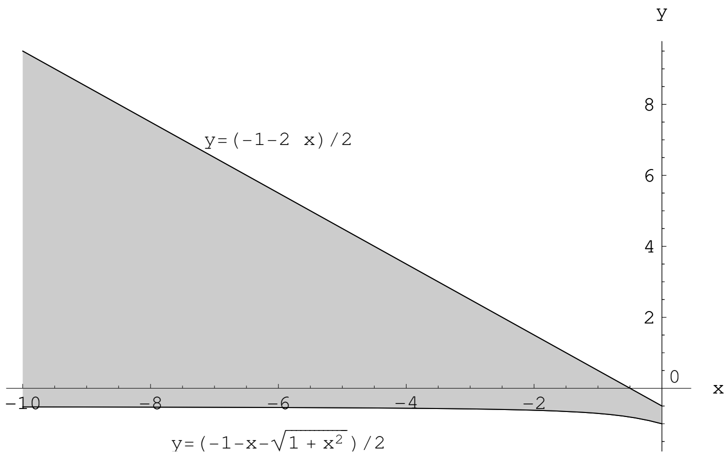

Figure 2: CILF Sufficient Existence Region

For the case of discrete-time systems, QE method is also applicable by using the following theorem from [10]:

Theorem 6: Given a nonsingular matrix $\mathbf{U} \in R^{n\times n}$, a necessary and sufficient condition for the existence of $\mathbf{D} = \mathrm{diag}\{d_1, ..., d_n\}$ ($d_i > 0, i = 1, ..., n$) such that $V(\mathbf{x}) = ||\mathbf{D^{-1}Ux}||_\infty$ to be a CILF for the set of systems (2) is that the simultaneous inequalities $\mathbf{B}_{di}\mathbf{d} > 0$ ($i = 1, ..., q; \mathbf{d} = \{d_1, ..., d_n\}^T \in \mathcal{R}^n$) have a common solution $\mathbf{d} > 0$, where $\mathbf{B}_{di} = \{b^i_{djk}\}$ is defined by

$$\begin{cases} b^i_{djj} = -|\tilde{a}^i_{djj}| + 1 \\ b^i_{djk} = -|\tilde{a}^i_{djk}|, (j \neq k), \end{cases} \tag{31}$$

and $\tilde{\mathbf{A}}_{di} = \{\tilde{a}^i_{djk}\} = \mathbf{U}^{-1}\mathbf{A}_{di}\mathbf{U}$.

Based on Theorem 6 and with similar steps as above, we can find symbolic sufficient CILF existence conditions for the systems (2).

The merit of QE method here is that it can give larger existence regions than by the methods in [10]. This is because conditions of these methods are implied by Theorem 5 and 6 to which QE method is directly applied.

4 Conclusion

In this paper, the problem of the application of QE method to the common Lyapunov function problem has been studied. The strength of the method lies in that it gives algebraic conditions in parameter space. It is shown that for a pair of second-order systems, QE method can give a symbolic existence condition of a CQLF. For a set of second-order systems, QE method is also applicable by combining the conditions in Theorem 3 and 4. For the CILF problem, it is difficult to apply QE method directly to the conditions in Definition 3, 4 due to intensive computation and complexity. But by decomposing $\mathbf{W} = \mathbf{D}^{-1}\mathbf{U}$ and utilizing sufficient conditions in Theorem 5 and 6, symbolic sufficient existence conditions are obtainable. Although QE method is applicable to the symbolic existence problem of common Lyapunov function, due to intensive computation, the solvable problems are limited by the numbers of system parameter and system order. Currently, for CQLF, the solvable problems are limited to a pair of second-order systems with two system parameters and a set of second-order systems with a single system parameter. For CILF, only sufficient conditions for a pair of second-order systems with two system parameters can be obtained. In order to solve more complicated problems, we need new Lyapunov function existence conditions with less variables and better QE algorithms together with stronger computation systems.

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